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SHORT NOTES

Discontinuous fault zones

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Abstract—Many tectonic faults and tension fractures are, at least initially, composed of separate segments. This note deals with a little explored reason for this phenomenon which, in faulting, has obvious implications both for the migration of hydrocarbons and for the sealing capacity of faults. Theoretical arguments based on Coulomb–Mohr's theory of shear failure and on a theorem for the integrability of vector fields lead to the expectation that, in general, non-uniform and truly three-dimensional stress fields will impede the formation of smooth, coherent fault surfaces; this is in contrast to the stress fields that are associated with plane deformation. Examples are given and special attention is drawn to the role of tectonic stress fields with horizontal principal stresses that change with depth in magnitude and direction.

INTRODUCTION

A PAPER by Segall & Pollard (1980), which deals with the elastic interaction of en-échelon shear cracks, opens with the statement that "faults are discontinuous geological features consisting of numerous discrete segments" and continues later with "the processes responsible for the formation of discontinuous faults are largely unknown . . ." Although the first statement may seem somewhat questionable in its generality (and probably is mainly applicable to faults in an early state of development) it may stress the importance of the subject and draw our attention to its relevance for hydrocarbon migration. The second statement should make it sufficiently clear that this note will be limited to certain aspects of the complex subject.

When a fault is being initiated, shearing deformation is concentrated in narrow domains. Usually this does not immediately lead to the formation of a through-going shear band, since inhomogeneities in rock strength and local variations of the stress field may promote early formation of shear-band segments which later have to be connected into a tectonic fault.

For example, bending of a sequence of alternating competent and incompetent (i.e. more ductile) beds will first induce faulting in the competent, brittle beds, while the incompetent beds will accommodate a certain amount of strain by creep or numerous small slips before being offset by faulting. Connection of the fault segments will then result in faults which, at least in an early state, have a tortuous and splintered shape.

Perhaps even more important than variations in lithology for the segmentary development of faults are variations in the stress field. It has, for instance, been mentioned by Beach (1975) that faults, when propagating into a region with differently oriented principal stresses, will adjust to the change in stress regime by breaking up into an en-échelon array of smaller shears. However, this I believe to be only one manifestation of a more general feature of certain non-uniform stress fields, which so far has not been noticed in rock mechanics and tectonics, and which is the subject of this note.

Concentrating on the effect of the tectonic stress field, I assume mechanical homogeneity and strength isotropy of the rock and consider faulting in the brittle regime. The orientation of tangential elements of faults or fault segments is then solely determined by the stress field that initiated faulting. I specify the orientation of these tangential elements further by Coulomb–Mohr's shear failure criterion, according to which the unit normal $\mathbf{n}$ of such an element is orthogonal to the axis of the intermediate principal stress and makes the angle $\mu = 45° + \varphi/2$ with the axis of the greatest compressive principal stress. The 'angle of internal friction' φ is assumed to be constant in the range of mean effective normal stresses considered.

The important question now arises: will the Coulomb-slip elements, associated with a stress system in the limit state, integrate into smooth, coherent slip surfaces? In other words, will the infinitesimal, planar Coulomb-elements form the tangential elements of sets of smooth surfaces? Or, applied to tectonic faulting, will the stress field in the limiting state allow or impede the inception of smooth, coherent fault surfaces?

To investigate this question I shall apply the so-called 'integrability theorem' for vector fields.

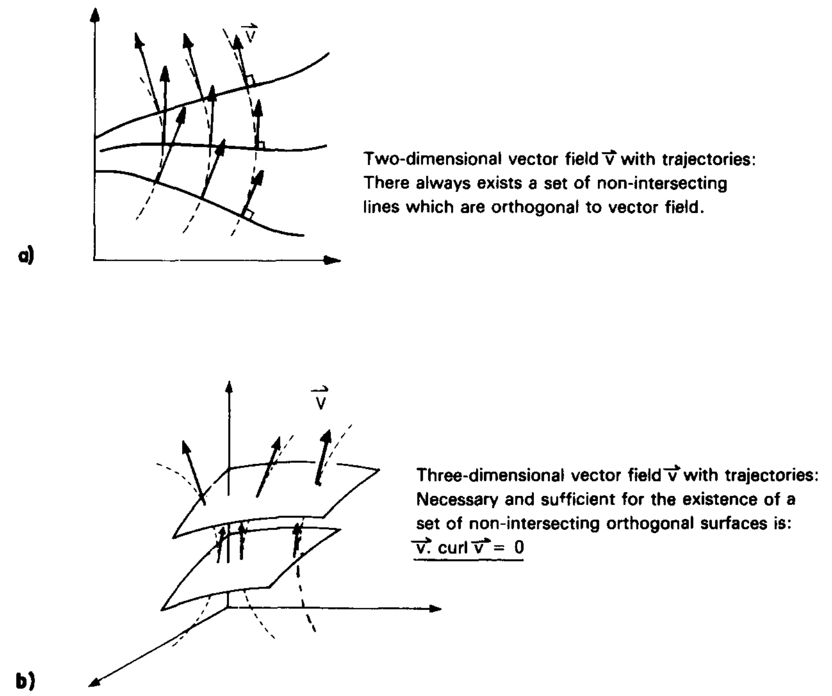


Fig. 1. Vector fields of principal stresses in two (a) and three (b) dimensions.

THE INTEGRABILITY THEOREM

The theorem states (see Appendix) that a differentiable vector field **v** will be everywhere normal to a set of smooth surfaces (Fig. 1) if, and only if, the scalar product of the vector field with its own curl vanishes

$$\mathbf{v}\cdot \mathrm{curl}\,\mathbf{v} = 0. \tag{1}$$

Written *in extenso* for a Cartesian *x*-, *y*-, *z*-system this condition becomes

$$\begin{aligned} &v_x\cdot(\mathrm{curl}\,\mathbf{v})_x + v_y\cdot(\mathrm{curl}\,\mathbf{v})_y + v_z\cdot(\mathrm{curl}\,\mathbf{v})_z \\ &\quad \equiv v_x(\partial v_z/\partial y - \partial v_y/\partial z) + v_y(\partial v_x/\partial z - \partial v_z/\partial x) \\ &\quad\quad + v_z(\partial v_y/\partial x - \partial v_x/\partial y) = 0. \end{aligned} \tag{2}$$

Now we identify the vector **v** with the unit normal **n** of a set of Coulomb-slip elements and consider the unit vectors $\mathbf{e}_{\mathrm{I}}$ and $\mathbf{e}_{\mathrm{III}}$ in the directions of the maximum and minimum (compressive) principal stresses. The vectors $\mathbf{n}$, $\mathbf{e}_{\mathrm{I}}$, $\mathbf{e}_{\mathrm{III}}$ are coplanar, since **n** is orthogonal to $\mathbf{e}_{\mathrm{II}}$, the direction of the intermediate principal stress. The three coplanar vectors are related by

$$\mathbf{n} = \mathbf{e}_{\mathrm{I}} \cos\mu + \mathbf{e}_{\mathrm{III}} \sin\mu, \tag{3}$$

where

$$\mu = 45° + \varphi/2 \tag{4}$$

is the angle between **n** and $\mathbf{e}_{\mathrm{I}}$.

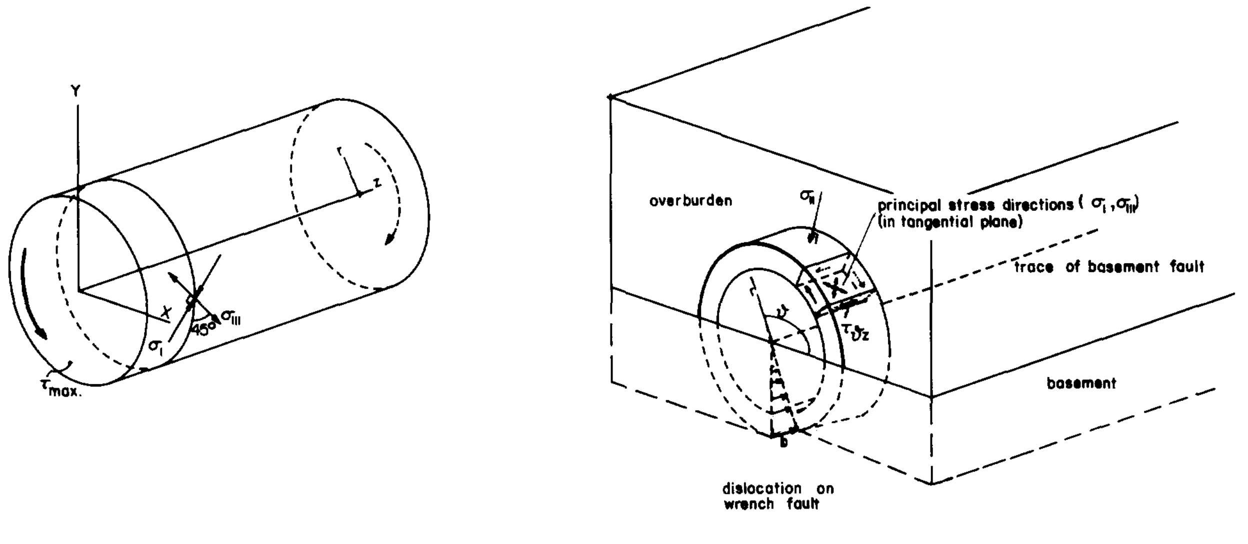
Since we have assumed that the material has the same angle of internal friction everywhere, application of eqn (1) to the vector field **n** yields the following condition for the existence of continuous and smooth slip surfaces

$$\begin{aligned} &\cos^2\mu\,\mathbf{e}_{\mathrm{I}}\cdot \mathrm{curl}\,\mathbf{e}_{\mathrm{I}} + \sin^2\mu\,\mathbf{e}_{\mathrm{III}}\cdot \mathrm{curl}\,\mathbf{e}_{\mathrm{III}} \\ &\quad + \sin\mu\cos\mu[\mathbf{e}_{\mathrm{I}}\cdot \mathrm{curl}\,\mathbf{e}_{\mathrm{III}} + \mathbf{e}_{\mathrm{III}}\cdot \mathrm{curl}\,\mathbf{e}_{\mathrm{I}}] = 0. \end{aligned} \tag{5}$$

This quasilinear partial differential equation of first order in the derivatives must thus be satisfied by the vector fields $\mathbf{e}_{\mathrm{I}}$ and $\mathbf{e}_{\mathrm{III}}$, if Coulomb-slips are to define continuous, smooth slip surfaces.

It is now easily seen that this condition is always satisfied if the directions of the greatest and smallest principal stresses are everywhere parallel to a given plane, as in problems of plane stress, and do not vary in a direction orthogonal to this plane. Identifying this plane with the *x*-, *y*-plane, all derivatives in the *z*-direction vanish and, from the definition of curl **v** (see eqn 2), it follows that the only non-vanishing component of curl $\mathbf{e}_{\mathrm{I}}$ and curl $\mathbf{e}_{\mathrm{III}}$ is the *z*-component. Since, moreover, the *z*-components of $\mathbf{e}_{\mathrm{I}}$ and $\mathbf{e}_{\mathrm{III}}$ vanish, each of the inner products in eqn (5) vanishes and the integrability condition is satisfied. Hence, in an isotropic, uniform material in a state of *plane strain* or *plane stress*, there will always exist a set of surfaces the normals of which coincide with the normals (eqn 3) of Coulomb-slip elements (Fig. 1b). In other words, the stress field will not impede the formation of *continuous, smooth fault surfaces*.

However, the situation is drastically changed when the *stress field is truly three-dimensional* in the sense that no Cartesian reference frame can be found such that *z*-components and/or *z*-derivatives of the unit vectors $\mathbf{e}_{\mathrm{I}}$ and $\mathbf{e}_{\mathrm{III}}$ vanish. The products in condition (5) will then no longer vanish separately, and the condition will impose a genuine and severe constraint on direction fields of the maximum and minimum principal stresses that are compatible with the existence of coherent, smooth slip surfaces. Therefore, within the framework of Coulomb–Mohr's failure theory, *incipient faults are, in general, not expected to form continuous, smooth surfaces in three-dimensional stress fields*. This may be illustrated by a few examples of three-dimensional stress fields which violate the integrability condition (5).



a. TORSION OF CIRCULAR ELASTIC CYLINDER:
Maximum shear stress acting parallel to circumference upon transverse cross-sections. Hence, highest tensile stress (σ_{III}) inclined at 45° to cylinder axis and parallel to surface.

b. SCREW DISLOCATION AS MODEL OF INITIAL STRESS FIELD IN OVERBURDEN NEAR BASEMENT FAULT.
($\tau_{z\vartheta} = \tau_{\vartheta z} \sim b/r$ is maximum shear stress; hence σ_{III} inclined at 45° to cylinder axis)

Fig. 2. Three-dimensional fields of principal stress vectors.

THREE-DIMENSIONAL TECTONIC STRESS FIELDS

Figure 2(a) shows the case of small, pure torsion of a circular cylindrical bar, the axis of which coincides with the z-axis. The planar cross-sections perpendicular to the cylinder axis are subject to the maximum shear stress which acts in the circumferential direction. Consequently, the greatest (compressive) and smallest (tensile) principal stresses act tangential to the curved cylinder surface at an inclination of $\pm 45°$ to the cylinder axis. One may easily verify that the associated unit vectors $\mathbf{e}_I$, $\mathbf{e}_{III}$ have the same x- and y-components, but their z-components have opposite signs

$$v_x = y/r\sqrt{2},\ v_y = -x/r\sqrt{2},\ v_z = \pm 1/\sqrt{2}. \qquad (6)$$

The inner product of the two vectors therefore vanishes, in agreement with their orthogonality. With reference to eqn (2) one notices that only the z-components of the curls of $\mathbf{e}_I$ and $\mathbf{e}_{III}$ do not vanish and attain the same value $(-1/r\sqrt{2})$. Inserting the components (6) in eqn (5) one finds that the left-hand side becomes $(\cos^2 \mu - \sin^2 \mu)/2r$. Because of eqn (4) this value differs from zero, and condition (5) is not satisfied. In fact, the deviation from zero increases in magnitude as the distance from the cylinder axis decreases. The case of a stationary screw dislocation (shown in Fig. 2b) is very similar. The similarity is apparent when the elements of a hollow cylinder are considered, the axis of which coincides with the dislocation line. The dislocation slip has occurred on a vertical plane over the triangular area with base b. If we interpret this dislocation as slip on a basement wrench fault and allow basement and overburden to have the same elastic properties, the stress field associated with the dislocation will represent the response in the overburden to incipient wrench faulting in the basement. Again, as in the case of torsion, condition (5) is violated, which strongly suggests that Coulomb slip surfaces ('Riedel' faults), which form when the stresses reach the limiting state should, at least initially, consist of incoherent segments in the vicinity of the basement fault.

A further example, inspired by a paper by McGarr (1980), is shown in Fig. 3. Here two horizontal principal stresses, σ_I and σ_{III}, of a regionally uniform tectonic stress system vary with depth, both in magnitude and direction. This state of stress may be viewed genetically as resulting from the superposition of two more elementary regional states of stress which reflect two distinct tectonic phases. The first phase has established a state of stress (marked by a superscript (1)) such that the principal stress difference decreases linearly with depth

$$\sigma_I^{(1)} - \sigma_{III}^{(1)} = \sigma_x^{(1)} - \sigma_y^{(1)} = a - bz. \qquad (7)$$

This may, for instance, have taken place during regional uplift of an elongated basin. In a second tectonic phase (indicated by the superscript (2) in Fig. 3) a horizontal compression is produced with horizontal principal stresses $\sigma_I^{(2)}$ and $\sigma_{III}^{(2)}$, constant over the depth range considered, and acting at 45° to the principal stress directions of the first tectonic phase. The x- and y-planes are then planes of maximum shear stress of the second system. The shear stress $\tau_{xy}^{(2)}$ and the normal stresses $\sigma_x^{(2)} = \sigma_y^{(2)}$ on these planes can then immediately be expressed in terms of the principal stresses of system (2) as stated in the figure. Assuming linear elastic behaviour of the rock mass during the compression phase (2), we may add

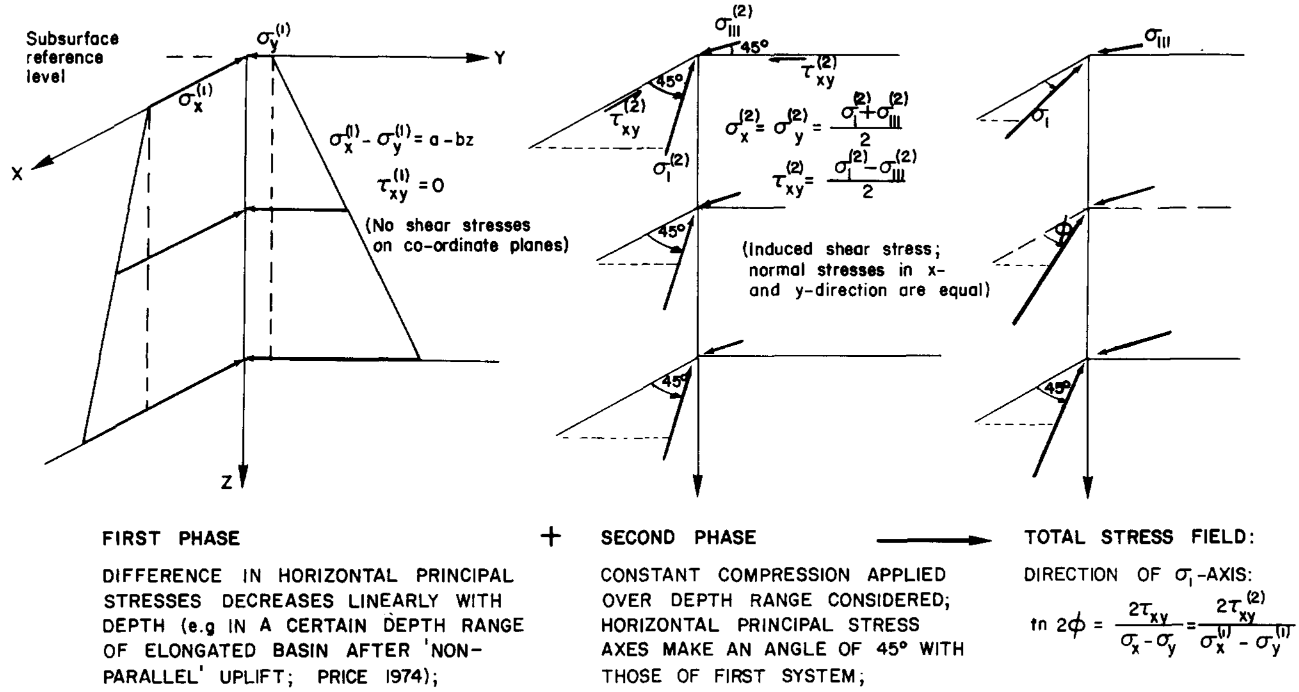


Fig. 3. Superposition of two regional stress regimes whose principal stresses do not coincide. (The vertical stress is a principal stress.)

corresponding components of the two stress fields. When the stress components σ_x, σ_y, τ_{xy} have been obtained, the angle Φ the σ_I-axis makes with the positive x-axis may be determined by an elementary formula as indicated in the figure. Its value becomes the following function of the depth coordinate z

$$\Phi = \tfrac{1}{2}\arctan\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \tfrac{1}{2}\arctan\frac{2\tau_{xy}}{\sigma_x^{(1)} - \sigma_y^{(1)}}$$
$$= \tfrac{1}{2}\arctan\frac{\sigma_I^{(2)} - \sigma_{III}^{(2)}}{a - bz}. \qquad (8)$$

Note that the relevant principal stress axes of the combined system are again horizontal, the intermediate principal stress remaining vertical, and that therefore the z-components of the unit vectors $\mathbf{e}_I$, $\mathbf{e}_{III}$ vanish. However, these vectors rotate with depth and, consequently, the z-derivatives in the curl components (2) will not vanish. The components of the two unit vectors are

$$e_{Ix} = \cos\Phi, \quad e_{Iy} = \sin\Phi, \quad e_{Iz} = 0$$
$$e_{IIIx} = -\sin\Phi, \quad e_{IIIy} = \cos\Phi, \quad e_{IIIz} = 0. \qquad (9)$$

With these components inserted, the left-hand side of condition (5) turns out to be equal to the magnitude of the gradient $d\Phi/dz$ of the rotation angle Φ which, in view of (8), does not vanish as long as $b \neq 0$ and the principal stresses of the compressive tectonic phase (2) differ in magnitude.

The last example may stand for many more complex tectonic situations where the horizontal stresses are anisotropic and the differential stress varies with depth.

A change in horizontal principal stress difference may take place, for instance, across the interface of beds which differ in stress response to the same regional strain. A second tectonic phase of the type shown in Fig. 3 will then also cause the directions of the bed-parallel principal stresses to change discontinuously across the interface. Faulting across such an interface will therefore be complicated by an abrupt change in strike direction.

The compressive second phase in Fig. 3 may be produced by a uniaxial tectonic compression or by a (sinistral) horizontal simple shear that does not affect the role of the overburden stress as intermediate principal stress, nor introduce a dependence of the stresses on x and y. When, in the simple shear case, the resultant stresses are in the limiting state conducive to wrench faulting, potential fault elements will be vertical and their strike line will be straight. The strike direction, however, will change with depth because of the changes in the direction of the horizontal principal stresses. It is obvious that the twisted elements cannot combine into a coherent smooth vertical fault surface, the incoherence becoming more noticeable the wider the fault zone extends along strike. This incoherence of wrench faults in the vertical direction should not be confused with the well-known en-échelon array of such faults ('Riedel' shears) in plan view.

Considering somewhat modified versions of the situation depicted in Fig. 3, we may, for instance, assume that the vertical normal stress is the maximum compressive stress and remains so during subsequent tectonic loading (phase 2), that is the principal stress orientation associated with normal faulting. Assuming that the state of stress is already close to the limiting state, a relatively small horizontal simple shear along the y-, z-plane may sufficiently reduce a horizontal normal stress, without affecting the maximum principal stress character of the overburden stress, to induce faulting. Because of vari-

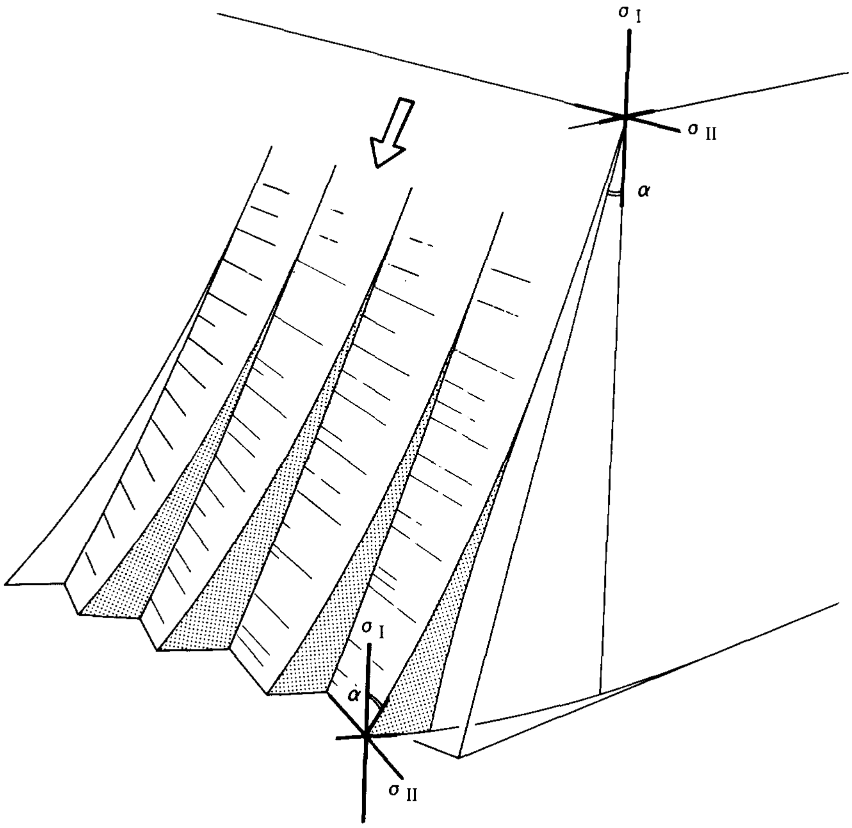


Fig. 4. Segmentation of a normal fault in a stress field in which horizontal principal directions rotate with depth.

ations of lithology and confining pressure with depth, the stress changes imposed by this shearing phase, in particular $\tau_{xy}^{(2)}$, will, in general, vary with depth. This and/or the depth-dependence of the initial horizontal differential stress will again make the orientation angle Φ of the resulting horizontal minimum compressive stress a function of z. The unit vector $\mathbf{e}_{III}$ associated with this principal stress has the components stated in (9), while the unit vector $\mathbf{e}_{I}$ associated with the vertical maximum principal stress has the components 0, 0, 1. Inserting for these vectors in (5) one easily verifies that all terms vanish with the exception of $\mathbf{e}_{III} \cdot \text{curl } \mathbf{e}_{III} \neq 0$, because $d\Phi/dz \neq 0$. The incipient normal faults will therefore consist of segments which may have the same dip angle, but which will vary in strike and slip direction.

A rather similar result follows when instead of a simple-shear phase 2, a uniaxial extension is imposed in a direction markedly different from the initial orientation of the smallest compressive stress. A normal fault entering a depth interval with rotated horizontal principal stresses is therefore expected to break up into segments as sketched in Fig. 4.

CONCLUSIONS

In view of the cases discussed above, it would seem rather exceptional for a non-uniform stress field with non-planar character, or not associated with plane deformation, to satisfy the integrability condition (5). And even if such a truly three-dimensional stress field were found to satisfy this condition, a slight variation of the field, produced by a minor change in boundary conditions, would immediately cause violation of condition (5). Thereby, it will hardly matter whether or not the stress fields are subject to the additional constraint of the limiting condition, required for the initiation of Coulomb-slip elements.

Hence, we conclude on the basis of Coulomb–Mohr's theory of shear failure, that in addition to variations in lithology (e.g. sand–shale sequences), non-uniformities of truly three-dimensional stress fields are a main cause of segmentary fault development. In such stress fields, it will be the rule, rather than the exception, that incipient faults consist of separate segments which do not lie on a smooth surface. This holds, in particular, for tectonic stress fields with horizontal principal stresses that change with depth in magnitude and direction.

In contrast with this, the continuity of an incipient fault, in a lithologically uniform material, is not affected by non-uniformities of the stress field, if the deformation or the stresses are of a strictly planar type.

The segmentation of incipient faults, which may be difficult to detect on seismic records, will not only allow across-fault migration of liquids, but also pre-set the width of the disturbed zone surrounding a fully developed fault.

Although this paper is concerned with the continuity or discontinuity of shear fractures, it should be noted that rather similar considerations apply to macroscopic extension fractures, that is extension fractures that are large when compared with the size of the textural inhomogeneities of the rock material. It is generally accepted that such fractures develop normal to the direction of the smallest compressive (or greatest tensile) stress σ_{III}. Therefore, when applying condition (1), the vector **v** has to be identified with the unit vector $\mathbf{e}_{III}$ and eqn (5) is then replaced by the simpler relationship

$$\mathbf{e}_{III}\cdot\text{curl }\mathbf{e}_{III} = 0. \tag{5a}$$

If this condition is not satisfied by the direction field of the principal stress σ_{III}, macroscopic extension fractures cannot develop as continuous fractures. Again, the condition is always satisfied by planar stress fields and in plane strain, but it will rarely be satisfied by non-uniform, truly three-dimensional stress fields, and in particular not by any of the stress fields discussed in the preceding section. A diagram illustrating the break-up of a smooth extension crack into segments can be found in a paper by Pollard *et al.* (1982). In the laboratory, Sommer (1969) has demonstrated the disintegration of a smooth parent crack into numerous fracture 'lances', by hydraulically fracturing a glass rod under torsion. Indeed, in 1930, in a paper by Lagalli on the mechanics of crevasses in glaciers, the non-uniformity of the three-dimensional stress field was recognized as a cause of the discontinuity of extension fractures. Unfortunately, Lagalli's paper has been widely ignored by modern glaciologists.*

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* For instance, in Paterson's *Physics of Glaciers*, Oxford University Press, 2nd Edn. 1981, in 20 pages of reference not a single reference can be found to the glaciological journal in which Lagalli had published.

APPENDIX

When is a vector field $\mathbf{v}(x_1, x_2, x_3)$ at any point of a three-dimensional region normal to a surface passing through that point and belonging to a family of non-intersecting smooth surfaces? Let $\Psi(x_1, x_2, x_3) = c$ represent the family of surfaces (c being a parameter), then the gradient grad Ψ with Cartesian coordinates $\partial\Psi/\partial x_1$, $\partial\Psi/\partial x_2$, $\partial\Psi/\partial x_3$ is a vector field which, at all points, is normal to one of these surfaces, and our question may be rephrased: when does a scalar field $\Psi(x_1, x_2, x_3)$ exist such that

$$\mu\mathbf{v} = \text{grad }\Psi, \tag{10}$$

where the proportionality factor μ is a scalar function of the space coordinates? A completely equivalent expression is obtained by scalar multiplication of eqn (10) with the infinitesimal vector $\mathbf{dx} \equiv (\mathrm{d}x_1, \mathrm{d}x_2, \mathrm{d}x_3)$

$$\mu\mathbf{v}\cdot\mathbf{dx} = \mathrm{d}\Psi, \tag{10a}$$

where $\mathrm{d}\Psi$ is a total differential.

One may now easily see that validity of eqn (10) implies that condition (1) is satisfied. Applying the curl-operator to eqn (10) and noting that the curl of a gradient always vanishes, we obtain

$$\text{curl }\mu\mathbf{v} = \mu\text{ curl }\mathbf{v} + \text{grad }\mu \times \mathbf{v} = 0.$$

Since the vector associated with the vector (cross) product term is orthogonal to **v**, scalar multiplication of the last expression with **v** leads to

$$\mathbf{v}\cdot\text{curl }\mathbf{v} = 0. \tag{1}$$

This proves that eqn (1) is a *necessary* condition for the vector field **v** to be parallel to the normals of a family of smooth non-intersecting surfaces everywhere in the region considered (Fig. 1).

In fact, eqn (1) is also a *sufficient* condition as is shown in the theory of first-order linear differential equations ('Pfaff problem'). The proof, which may be found in textbooks on differential equations, demonstrates that the validity of eqn (2) implies that an 'integrating factor' μ can be found which makes expression (10a) a total differential. Interested geological readers may find a straightforward proof in Kestin's (1966; pp. 466–472) textbook of thermodynamics.